- 6. Carrington, T., and N. Davidson, J. Phys. Chem., 57, 418
- 7. Britton, D., N. Davidson, and G. Schott, Disc. Faraday Soc., 17, 58 (1954).
- 8. Britton, D., and N. Davidson, J. Chem. Phys., 23, 2461
- Greene, E. F., J. Am. Chem. Soc., 76, 2127 (1954).
   Glick, H. S., W. Squire, and A. Hertzberg, "Fifth Symposium International on Combustion," p. 393, Reinhold, New York (1955).
- 11. Glick, H. S., J. J. Klein, and W. Squire, J. Chem. Phys., 27, 850 (1957).
- 12. Palmer, H. B., and D. F. Hornig, ibid., 26, 98 (1958).
- 13. Glick, H. S., "Seventh Symposium (International) on Combustion," p. 98, Butterworth Scientific Publications, London, England (1959).
- 14. Greene, E. F., R. L. Taylor, and W. L. Patterson, Jr., J. Phys. Chem., 62, 238 (1958).
- 15. Skinner, G. B., and R. A. Ruehrwein, ibid., 63, 1736
- 16. Skinner, G. B., and W. E. Ball, ibid., 64, 1025 (1960).
- 17. Skinner, G. B., and E. M. Sokoloski, *ibid.*, 1028, 1052. 18. Kervorkian, V., C. E. Heath, and M. Boudart, *ibid.*, 964.
- 19. Britton, D., ibid., 742.
- 20. ——, and R. M. Cole, *ibid.*, **65**, 1302 (1961). 21. Miller, I., and S. W. Churchill, *A.I.Ch.E. Journal*, **8**, 201 (1962).

- 22. Weatherford, W. D., Paper presented at the 55th Annual Meeting of the A.I.Ch.E. (December, 1962).
  23. Jacobs, T. A., and R. R. Giedt, *ibid*.
  24. Gluckstein, M. E., Ph.D. thesis, Univ. Mich., Ann Arbor,
- Michigan (1957).
- 25. Roshko, A., Phys. Fluids, 3, 835 (1960).
- 26. "Selected Values of Chemical and Thermodynamic Properties," U.S. Nat. Bur. Stds., Circular 500 U.S. Government Printing Office, Washington, D. C., (1952).
- 27. Farkas, A., and L. Farkas, Proc. Roy. Soc. (London). A152, 124 (1935).
- 28. Farkas, A., "Ortho-Hydrogen, Para-Hydrogen and Heavy Hydrogen," Cambridge University Press, Cambridge, England (1935).
- 29. Farkas, L., and E. Wigner, Trans. Faraday Soc., 32, 708 (1936).
- 30. Van Meersche, M., Bull. Societe Chemie de Belgique, 60, 99 (1951)
- 31. Roginsky, S. Z., "Theoretical Principles of Isotope Methods for Investigating Chemical Reactions," Academy of Sciences USSR Press, Moscow (1956), translated by Con-
- sultants Bureau, Inc., AEC-tr-2873.
  32. Snow, R. H., R. E. Peck and C. G. Von Fredersdorff, A.I.Ch.E. Journal, 5, 304 (1959).

Manuscript received August 3, 1959; revision received June 26, 1963; paper accepted July 1, 1963. Paper presented at A.I.Ch.E. Kansas City meeting.

# A Mathematical Solution for the Condensation of Vapors from Noncondensing Gases in Laminar Flow inside Vertical Cylinders

### WILLIAM D. BAASEL and JULIAN C. SMITH

The basic method of solution, although presented for condensation, also is applicable to other mass transfer

systems such as wetted-wall columns, dissolution from

pipes of soluble material, and gas absorption. Because it

may be used for such widely diverse systems and because

it estimates the concentration and velocity profiles as well as the amount of mass transferred, this solution probably may be used to check the validity of the following assump-

tions used in the mathematical solution:

1. Operation was at steady state conditions.

Cornell University, Ithaca, New York

Previous efforts (2, 3, 8, 9, 11) to predict the rate of condensation from noncondensing gases in a cylindrical tube have been successful with a turbulent main flow. However, with laminar main flow the predicted rates often differ widely from the measured rates. These efforts are based on the macroscopic approach, frequently called film

This paper presents a fundamental approach to the problem. The method used may be compared with the Graetz solution for heat transfer and becomes the Graetz solution when there is no condensation. Graetz made certain simplifying assumptions and used the microscopic approach to predict the rate of heat transfer. The solution presented herein applies the microscopic approach to mass transfer. It permits the determination of the rate of condensation if the physical constants and the dimensions of the system are known and the main flow is laminar.

2. The fluids were Newtonian and isotropic. 3. The flow pattern inside the vertical cylinder had rotational symmetry. 4. The molar density and diffusivity of the gases were

constant throughout the tube. (The former is true at constant temperature and pressure for an ideal gas.)

5. No supersaturation occurred in the condensing sec-

William D. Baasel is with Ohio University, Athens, Ohio.

- 6. All condensation occurred at the gas-liquid or gassolid interface.
- 7. The calming section was long enough to allow a parabolic velocity profile at the entrance to the condenser.
- 8. The velocity at the gas-liquid or gas-solid interface was negligibly small in comparison with the average vel-
- ocity of the gas.
  9. The condensate thickness was negligibly small when compared to the diameter of the tube.
- 10. The temperatures of the wall and of the gas upstream from the condenser entrance were the same and were above the dew point of the gas.
- 11. The temperature at the gas-liquid and gas-solid interface in the condensing section was constant.
- 12. In the condensing section, there was one-way diffusion perpendicular to the wall. There was no net transfer of the inert gas perpendicular to the wall.
  - 13. Convection currents were negligible.
  - 14. The gas-liquid surface had no ripples.
- 15. The system may be treated as a binary mixture with one noncondensible and one condensible component.

This solution does not assume that the velocity distribution is parabolic throughout the condenser nor that any terms in the partial differential equations for the total material balance or the component balance may be ignored. Thus, this solution is more general than most previous solutions to problems of heat and mass transfer.

Condensation is somewhat similar to applying suction on the fluid at the wall. In the case of suction, the material drawn to the wall passes through the wall and is removed from the system. In condensation, the molecules brought to the wall are removed from the fluid by condensing them. In either case, when the velocity perpendicular to the wall is determined at the wall and integrated over the surface of the cylinder, the rate of removal of material from the system may be calculated. If  $N_A$  is the number of moles transferred in an area, A, then

$$\frac{dN_A}{dA} = -D_{AB} \left(\frac{\partial C_A}{\partial r}\right)_w + \left(\frac{dN_A}{dA} + \frac{dN_B}{dA}\right) \frac{C_{AW}}{C_T} \tag{1}$$

In the case of condensation  $dN_B$  must be zero at the wall. Otherwise, some of the noncondensible gas would either pass through the wall or condense, both of which are impossible. To give the amount condensed, Equation (1) is rearranged to

$$dN_A = -\frac{C_T D_{AB}}{C_T - C_{AW}} \left(\frac{\partial C_A}{\partial r}\right)_{vv} dA \tag{2}$$

and then integrated over the length of the condenser to give

$$N_A = -\frac{C_T D_{AB} D_{\pi}}{C_T - C_{AW}} \int_0^L \left(\frac{\partial C_A}{\partial r}\right)_{to} dx \tag{3}$$

The only unknown in this equation,  $(\partial C_A/\partial r)_w$ , may easily be calculated if CA is determined as a function of x and r. To do this, the continuity equation, the Navier-Stokes equation, the energy balance equation, and the component balance equation must be solved simultaneously.

To solve these four equations certain assumptions were made. One was to assume constant temperature; this eliminated the energy-balance equation. Also, since the velocity of the noncondensible component perpendicular to the wall was zero at the wall, it was assumed that this velocity was small elsewhere. If a long condenser were used so that equilibrium conditions existed at the exit of the condensing section, then the velocity distribution normal to the wall would be parabolic both at the entrance and exit of the condenser. Also, for a long condenser the

velocity normal to the wall of the noncondensible component at any radial distance from the wall must be the same at the entrance and exit of the condenser. Thus the net velocity of the noncondensible component perpendicular to the wall must be zero. Therefore the assumption that the point velocity of the noncondensible component is zero everywhere is a reasonable simplification. This does not mean that the velocity perpendicular to the wall of the condensible component is zero. If this were so, there would be no condensation and no problem to solve. Therefore this solution differs from most previous solutions for heat or mass transfer in that the velocity perpendicular to the wall is not assumed to equal zero and the velocity parallel to the wall is not assumed to be parabolic in the condensing section.

The net flow in the radial direction for one-way diffusion

$$\frac{dN_A}{dA} = -\frac{D_{AB} C_T}{(C_T - C_A)} \frac{\partial C_A}{\partial r} \tag{4}$$

Since the only net flow in the radial direction is due to the diffusing component

$$\frac{dN_A}{dA} = vC_T \tag{5}$$

When Equation (5) is substituted into Equation (4), one obtains

$$v = -\frac{D_{AB}}{C_T - C_A} \frac{\partial C_A}{\partial r} \tag{6}$$

From assumption 12 we obtained v as a function of the concentration, and because of this assumption the Navier-Stokes equation may be eliminated. The other equations that must be solved simultaneously with Equation (6) are the continuity and component-balance equations. These are, respectively

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \tag{7}$$

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial r} = D_{AB} \left( \frac{\partial^2 C_A}{\partial r^2} + \frac{1}{r} \frac{\partial C_A}{\partial r} + \frac{\partial^2 C_A}{\partial x^2} \right)$$
(8)

Equations (7) and (8) are not mutually independent.

The boundary conditions that must be satisfied by these

for 
$$x = 0, u = u_{mo} \left( 1 - \frac{r^2}{r_w^2} \right), \frac{\partial C_A}{\partial x} = 0, C_A = C_{AO}, v = 0$$

 $x = \infty, u = u_{mf} \left( 1 - \frac{r^2}{r_{vc}^2} \right), \frac{\partial C_A}{\partial x} = 0, C_A = C_{AW}, v = 0$ 

 $r = r_w, u = 0, \frac{\partial C_A}{\partial r} = 0, C_A = C_{AW}$ 

for r = 0,  $\partial u/\partial r = 0$ ,  $\partial C_A/\partial r = 0$ , v = 0

All theoretical equations must be independent of the units used, and the variables are used most conveniently in the form of dimensionless ratios. When  $C_{A'}$ , u', v', y, and s are substituted into Equations (6), (7), and (8), respectively one obtains

$$v' = -\frac{2 D_{AB}}{D u_{mo} (K - C_{A'})} \frac{\partial C_{A'}}{\partial s}$$
 (9)

$$\frac{\partial (su')}{\partial y} + \frac{\partial (sv')}{\partial s} = 0 \tag{10}$$

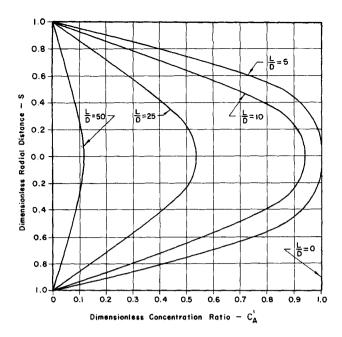


Fig. 1. Concentration profiles for K=2.0 and M=200 at various values of L/D.

$$u'\frac{\partial C_{A'}}{\partial y} + v'\frac{\partial C_{A'}}{\partial s} = \frac{2D_{AB}}{Du_{mo}} \left( \frac{\partial^2 C_{A'}}{\partial s^2} + \frac{\partial C_{A'}}{s\partial s} + \frac{\partial^2 C_{A'}}{\partial y^2} \right) \tag{11}$$

where  $K = (C_T - C_{AW})/(C_{AO} - C_{AW})$ .

Equation (9) conforms to all the boundary conditions placed on v.

Eliminating v' from Equations (10) and (11), using Equation (9), and then rearranging gives

$$\frac{\partial u'}{\partial y} = \frac{2D_{AB}}{Dsu_{mo}} \frac{\partial}{\partial s} \left( \frac{s}{K - C_{A'}} \frac{\partial C_{A'}}{\partial s} \right)$$
(12)

$$u' \frac{\partial C_{A'}}{\partial y} - \frac{2D_{AB}}{Du_{mo}} \frac{\partial^2 C_{A'}}{\partial y^2} = \frac{2D_{AB}}{Du_{mo}}$$

$$\left[\frac{\partial^2 C_{A'}}{\partial s^2} + \frac{1}{s} \frac{\partial C_{A'}}{\partial s} + \frac{1}{(K - C_{A'})} \left(\frac{\partial C_{A'}}{\partial s}\right)^2\right]$$
(13)

When both sides of Equation (13) are multiplied by  $s/(K-C_A')$ 

$$\left(u'\frac{\partial C_{A'}}{\partial y} - \frac{2D_{AB}}{Du_{mo}}\frac{\partial^{2}C_{A'}}{\partial y^{2}}\right)\frac{1}{K - C_{A'}} \\
= \frac{2D_{AB}}{Dsu_{mo}}\frac{\partial}{\partial s}\left(\frac{s}{K - C_{A'}}\frac{\partial C_{A'}}{\partial s}\right) \quad (14)$$

The right-hand sides of Equations (12) and (14) are equal. The left-hand sides must therefore be equivalent and may be set equal to each other:

$$\left(u'\frac{\partial C_{A'}}{\partial y} - \frac{2D_{AB}}{Du_{mo}}\frac{\partial^2 C_{A'}}{\partial y^2}\right)\frac{1}{K - C_{A'}} = \frac{\partial u'}{\partial y} \quad (15)$$

or

$$-\frac{\partial [u'(K-C_{A'})]}{\partial u} = \frac{2D_{AB}}{Du_{mo}} \frac{\partial^2 C_{A'}}{\partial u^2}$$
(16)

When Equation (16) is integrated, one obtains

$$u'(K - C_{A'}) = \frac{-2D_{AB}}{Du_{mo}} \frac{\partial C_{A'}}{\partial y} + K(s)$$
 (17)

where K(s) is the constant of integration which is a function of the radius. K(s) is evaluated with the boundary conditions given for  $x = \infty$  to give

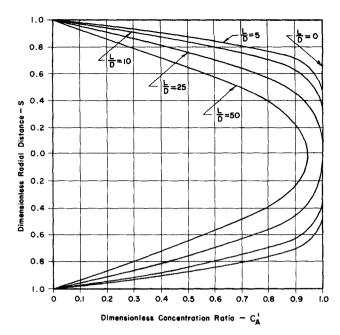


Fig. 2. Concentration profiles for K=2.0 and M=1,000 at various values of L/D.

$$K(s) = \frac{Ku_{mf}}{u_{mo}} (1 - s^2) \tag{18}$$

When this is substituted into Equation (17)

$$u' = -\frac{2D_{AB}}{u_{mo}D(K - C_{A'})} \frac{\partial C_{A'}}{\partial y} + \frac{u_{mf}K}{u_{mo}(K - C_{A'})} (1 - s^{2})$$
(19)

Equation (19) conforms to all the boundary conditions placed on u.

Equation (19) may now be substituted in either Equation (12) or Equation (14) to yield

$$-\frac{2D_{AB}}{u_{mo}D(K-C_{A'})} \left(\frac{\partial C_{A'}}{\partial y}\right)^{2} + \frac{u_{mf}}{u_{mo}} \frac{K}{K-C_{A'}} (1-s^{2}) \frac{\partial C_{A'}}{\partial y} - \frac{2D_{AB}}{Du_{mo}} \frac{\partial^{2}C_{A'}}{\partial y^{2}} = \frac{(K-C_{A'})2D_{AB}}{su_{mo}D} \frac{\partial}{\partial s} \left(\frac{s}{K-C_{A'}} \frac{C_{A'}}{s}\right)$$
(20)

When both sides are multiplied by  $[Du_{mo}/D_{AB}]$   $(K - C_{A'})$  one obtains

$$-\left(\frac{\partial C_{A'}}{\partial y}\right)^{2} + MK \left(1 - s^{2}\right) \frac{\partial C_{A'}}{\partial y}$$

$$-\left(K - C_{A'}\right) \frac{\partial^{2} C_{A'}}{\partial y^{2}} = \left(\frac{\partial C_{A'}}{\partial s}\right)^{2}$$

$$+ \frac{K - C_{A'}}{s} \frac{\partial C_{A'}}{\partial s} + \left(K - C_{A'}\right) \frac{\partial^{2} C_{A'}}{\partial s^{2}} \tag{21}$$

where  $M = u_{mf} D/2 D_{AB}$ .

The goal of this study was to solve Equation (3). In dimensionless terms Equation (3) becomes

$$N_A = -\frac{C_T D_{AB} D_{\pi}}{K} \int_0^{2L/D} \left( \frac{\partial C_{A'}}{\partial s} \right)_w dy \qquad (22)$$

The only unknown in Equation (22) is  $(\partial C_A'/\partial s)_w$  and may be calculated if  $C_A'$  is known as a function of y and s. Thus Equation (21) provides a solution since  $C_A'$  at any point with coordinates s and y is determined if M and K

are specified. M and K are constants which may be calculated easily for any condensation problem.

Attempts to solve Equation (21) analytically were unsuccessful, but it was solved with numerical approximations on a Burroughs-220 computer.

The concentration for given values of M and K was found at various values of s and y. (Figures 1 and 2). From this was obtained, for some actual runs, the value of

$$B(x) = -\pi \int_0^{2x/D} \left( \frac{\partial C_{A'}}{\partial s} \right)_w dy \qquad (23)$$

Some selected values of B(x) are given in Table 2. A more complete tabulation, tables of B(x) for values of K from 1.1 to 100 and  $(N_{Reo} N_{Sc})$  from 100 to 5,000, is given in reference 1.

The product of B(x) and  $C_TD_{AB}D/K$  gives an estimate of the total amount of material condensed. A comparison of this with experimental values found under the same conditions appears in Table 1.\*

The experimental data were obtained by Kusak (5), Griswold (4), and Unger (10) with a methanol and air system. The inlet Reynolds number was less than 2,000 for a total of thirty-five runs. Eighteen runs were for the gas flowing downward through the condenser, and seventeen were for the gas flowing upward.

For these runs, the temperature of the gas-condensate surface was not measured and had to be estimated so that the concentration at the interface could be calculated. The average temperature of the gas also had to be estimated to calculate an average molar density and diffusivity. These quantities were bracketed between two extremes. For the first estimate,  $A_1$ , the temperature of the gas-condensate surface was chosen as the average temperature of the cooling medium. The gas temperature was taken as the average of the entering gas temperature and the average temperature of the cooling medium.

For the other estimate,  $A_2$ , the interfacial temperature was taken as  $t_{cc3}$ , the temperature of the condensate shortly after it left the condenser. The gas temperature was calculated as the average of  $t_{cc3}$  and the inlet gas temperature. Both Kusak and Griswold mentioned that the fluctuations in  $t_{cc3}$  were "wide and abrupt." Because of this, Kusak did not always measure  $t_{cc3}$  and used  $t_{cc10}$  as a substitute.  $t_{cc10}$  was the temperature of the condensate after it left a holdup ring downstream of the condenser. Both Kusak and Griswold noted that the temperatures  $t_{cc3}$  and  $t_{cc10}$  were higher than the calculated value of the bulk condensate temperature, sometimes by as much as 20°C. Because of these fluctuations and the disagreement with calculated values, the first estimate, A<sub>1</sub>, was thought to be a more accurate estimate of actual conditions. In fact, the estimate of interfacial temperature by A1 would be correct if the thermal resistance of the cooling medium, the wall, and the condensate film could be neglected. Since the gas flow was in the laminar region throughout, this assumption was probably not greatly in error. For this reason comparisons between runs usually were made for the results obtained with estimate  $A_1$ . The estimated values given in Table I are for  $A_1$ .

For three of the thirty-five runs, the experimental results fell between the computer results obtained with  $A_1$  and  $A_2$ . For all other runs, the calculated condensation rate was smaller than the measured rate, and estimate  $A_1$  more closely approximated the experimental results. For each of the eleven downflow runs of Kusak, the ratio of the calculated rate of condensation to the measured rate was greater than 0.63 with estimate  $A_1$  and 0.50 with

Table 2. B(x)For  $N_{Ref} N_{Sc} = 200$ 

| $x/D \setminus K$ | 1.5         | 2.0                  | 5.0             | 10.0  | 100.0 |
|-------------------|-------------|----------------------|-----------------|-------|-------|
| 5                 | 144         | 114                  | 81              | 74    | 68    |
| 10                | 217         | 167                  | 117             | 105   | 97    |
| 20                | 315         | 234                  | 158             | 141   | 129   |
| 30                | 378         | 273                  | 178             | 160   | 145   |
| 40                | 417         | 294                  | 188             | 167   | 152   |
| 50                | 440         | 305                  | 193             | 171   | 155   |
| ∞                 | 472         | 314                  | 197             | 175   | 159   |
|                   |             | For N <sub>Ref</sub> | $N_{Sc} = 1,00$ | 0     |       |
| 5                 | 268         | 210                  | 165             | 153   | 144   |
| 10                | 420         | 328                  | 247             | 227   | 212   |
| 20                | 647         | 499                  | 365             | 334   | 310   |
| 30                | 826         | 631                  | 454             | 414   | 383   |
| 40                | 976         | 741                  | 526             | 479   | 443   |
| 50                | 1,107       | 836                  | 588             | 534   | 493   |
| $\infty$          | 2,360       | 1,570                | 984             | 874   | 795   |
|                   |             | For $N_{Ref}$        | $N_{Sc} = 5,00$ | 0     |       |
| 5                 | <b>46</b> 9 | 377                  | 293             | 272   | 255   |
| 10                | 751         | 595                  | 450             | 415   | 387   |
| 20                | 1,188       | 929                  | 686             | 630   | 587   |
| 30                | 1,545       | 1,199                | 875             | 803   | 746   |
| 40                |             | 1,433                | 1,039           | 951   | 884   |
| 50                |             | 1,643                | 1,185           | 1,084 | 1,007 |
| <b>∞</b>          | 11.800      | 7.850                | 4.920           | 4.370 | 3 980 |

estimate A<sub>2</sub>. For none of the seven downflow runs of Unger and Griswold was this ratio greater than 0.56. This ratio for Kusak's seventeen upflow runs varied from 0.74 to 0.37 if run 88, which seemed to be in error, was excluded.

The ratio of the predicted rate of condensation to the measured rate of condensation was generally nearer 1.0 for downflow runs for two major reasons. First, for upflow runs, the calming section was only 3 ft. long, but for downflow runs it was 5 ft. long. Second, for upflow runs the collection ring just upstream of the condenser had a smaller diameter than the tube and would have distorted the flow pattern. When K < 1.5, the ratios of the predicted to the measured rates of condensation are very low partially because the computer solution for K < 1.5 may be greatly (as much as 30%) in error since the Burroughs-220 computer was not fast enough and the approximate solution used could not be checked satisfactorily against a more rigorous and time consuming solution.

This is not surprising since K is a measure of the potential amount of condensation. K is the inverse ratio of the maximum amount which could be condensed divided by the amount of noncondensible component plus the amount of the condensible which could be condensed. A small K indicates a large potential for condensation, and for this case the higher derivatives neglected by the numerical approximations in the computer solution might be important. Therefore, when K is small, the greatest deviation between predicted and experimental results might be expected to occur. Neither this nor any correspondence between the concentration and the difference in the predicted and experimental results could be verified because of the scatter of the data.

Two other possible reasons for the discrepancies between predicted and measured results are that condensation may have begun in the calming section thus giving a longer condenser than indicated, and that there may have been ripples at the condensate surface. For the system of methanol and air there would be no sizeable convection currents.

However, with other systems convection currents, which in this solution are assumed to be negligible, may be important especially if the condensation occurs in a long

Tabular material has been deposited as document 7657 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for \$1.25 for photoprints or for 35-mm. microfilm.

tube. The conditions under which natural convection becomes important are presently unknown.

Linton and Sherwood (6) suggested that the Graetz solution for heat transfer may be adapted easily for mass transfer. This generally is invalid for the case of condensation since its application requires that the second term in Equation (8) be considered negligible. Although v may be small compared to u,  $\partial C_A/\partial r$  is generally much greater than  $\partial C_A/\partial x$ . The net result is that the first and second terms in Equation (8) are frequently of comparable magnitude. Thus the rate of condensation predicted by the Graetz solution (7) was less than the condensation rate predicted by this paper. For low values of K the rate of condensation predicted by the Graetz solution was less than a quarter of that predicted by the method herein proposed. Also when K was larger than 100, the two predictions were close. Since the predicted results in both cases were generally lower than the experimental results, the treatment presented in this paper is a better approximation than the Graetz solution.

The general method of solution used in this study is applicable to more than one process. It applies directly to gas absorption if the gas phase is controlling and if the equilibrium gas composition at the interface may be considered constant.

The solution may also be applied to wetted-wall columns and diffusion from pipes, cast of soluble material, into flowing streams. The only major modification necessary for the last two systems is to change the definition of the dimensionless concentration ratio  $C_A$ .

In all cases it must be remembered that this solution applies to a specific mathematical model. For industrial applications where there is no calming section and where natural convection currents may be important, this solution provides only a starting point for the development of design equations.

To account for variations from this mathematical model, two approaches may be used. One is to modify the mathematical-computer solution to allow for temperature variations, slip at the liquid-gas interface, etc. The other is to develop correction factors based on dynamic similarity to account for differences between the actual and ideal systems. The former is the best approach but may prove to be impractical when rippling and natural convection cannot be neglected.

The method presented in this paper with the appropriate modifications and correction factors should give far better design equations than are now available. Even without correction factors, tests designed specifically to check this solution are expected to give experimental results which fall within 25% of the calculated condensation rate.

## **ACKNOWLEDGMENT**

The authors wish to express their thanks to the staff of the Cornell Computing Center for their assistance in programming the Burroughs-220 computer.

#### NOTATION

$$A = \text{area of transfer, sq. ft.}$$

$$B(x) = -\pi \int_{0}^{2\pi/D} \left(\frac{\partial C_{A'}}{\partial s}\right)_{w} dy$$

 $C_A$ = concentration of condensible component A, lb. moles/cu. ft.

= dimensionless concentration ratio of the condensible component A,  $\frac{C_A - C_{AW}}{C_{AO} - C_{AW}}$ 

 $C_{AO}$  = concentration of condensible component A at the condenser inlet, lb. moles/cu. ft.

 $C_{AW}$  = concentration of condensible component A at the gas-condensate interface, lb. moles/cu. ft.

 $C_T$ = molar density, lb. moles/cu. ft.

= diameter of the condenser, ft.

 $D_{AB} = \text{diffusivity of } A \text{ in } B, \text{ sq. ft./hr.}$ K

 $= (C_T - \acute{C}_{AW})/(C_{AO} - \acute{C}_{AW})$ K(s) = constant of integration

length of the condenser, ft.

 $(u_{mf})(D/2)/(D_{AB})$ 

L

flow rate of condensible component A, lb. moles/

 $N_{AC}$  = amount of material condensed, lb. moles/hr.

 $N_{AC_c}$  = calculated rate of condensation, lb. moles/hr.

 $N_{AC_m}$  measured rate of condensation, lb. moles/hr.

 $N_{AO}$  = amount of condensible material which enters the condenser, lb. moles/hr.

flow rate of inert component, lb. moles/hr.  $N_B =$ 

Reynolds number,  $Du\rho/\mu$  $N_{Re} =$ 

 $N_{Ref}=Du_{mf}
ho/2\mu$ , Reynolds number, where  $x=\infty$   $N_{Reo}=Du_{mo}
ho/2\mu$ , Reynolds number at the entrance to the condenser

 $N_{Sc}$ = Schmidt number,  $\mu/\rho D_{AB}$ 

= radial distance from the center of the cylinder, ft.

= radius of the condenser, ft.  $r_w$ 

dimensionless radial distance,  $r/r_w$ 

temperature, °K. temperature, °F. Tt

= temperature of the gas-condensate surface, °F.  $t_c$ 

= temperature of the condensate shortly after leav $t_{cc3}$ ing the condenser, °C.

temperature of the condensate following  $t_{cc3}$ , °C.  $t_{cc10} =$ 

 $T_A$ average temperature of the gas stream °K.

molar average velocity of the gas parallel to the uwall, ft./hr.

dimensionless velocity parameter,  $u/u_{mo}$ u'

maximum velocity of the gas parallel to the wall  $u_{mf}$ at the exit of an infinitely long condenser, ft./hr.

maximum velocity of the gas parallel to the wall  $u_{mo}$ at the entrance to the condenser, ft./hr.

molar average velocity of the gas perpendicular to  $\boldsymbol{v}$ the wall, ft./hr.

dimensionless velocity parameter,  $v/u_{mo}$ v'

a subscript referring to conditions at the gasliquid interface

longitudinal distance from the condenser entrance, ft.

a dimensionless longitudinal distance, 2x/D

3.1416

w

= average viscosity of gas stream

average density of gas stream

## LITERATURE CITED

- 1. Baasel, W. D., Ph.D thesis, Cornell Univ., Ithaca, N. Y.
- Bras, G. H., Chem. Eng., 60, No. 4, p. 233 (1953); 60, No. 5, p. 238 (1953); 61, No. 5, p. 190 (1954).
- 3. Colburn, A. P., Trans. Am. Chem. Engrs., 29, 174 (1933).
- Griswold, H. E., M.S. thesis, Cornell Univ., Ithaca, N. Y. (1954)
- Kusak, L. J., Ph.D. thesis, Cornell Univ., Ithaca, N. Y. (1958)
- Linton, W. H., Jr., and T. K. Sherwood, Chem. Eng. Progr., **46**, 258 (1950).
- Sellers, J. R., M. Tribus, and J. S. Klein, Trans. Am. Soc. Mech. Engrs., 78, 441 (1956).
   Smith, J. C., Ind. Eng. Chem., 34, 1248 (1942); 34, 1495
- , and H. T. Robson, Paper presented at Am. Soc. Mech. Engrs. London Conference (1951).
- 10. Unger, E. W., Jr., M.S. thesis, Cornell Univ., Ithaca, N. Y.
- 11. Votta, F., Jr., and C. A. Walker, A.I.Ch.E. Journal, 4, 413 (1958).

Manuscript received June 13, 1962; revision received April 1, 1963; paper accepted April 5, 1963.